and

$$B = \frac{\mu_0 NI}{2\pi r} \quad \text{(within the toroid).} \tag{12.33}$$

The magnetic field is directed in the counterclockwise direction for the windings shown. When the current in the coils is reversed, the direction of the magnetic field also reverses.

The magnetic field inside a toroid is not uniform, as it varies inversely with the distance r from the axis *OO*'. However, if the central radius R (the radius midway between the inner and outer radii of the toroid) is much larger than the cross-sectional diameter of the coils r, the variation is fairly small, and the magnitude of the magnetic field may be calculated by **Equation 12.33** where $r = R$.

12.7 | Magnetism in Matter

Learning Objectives

By the end of this section, you will be able to:

- Classify magnetic materials as paramagnetic, diamagnetic, or ferromagnetic, based on their response to a magnetic field
- Sketch how magnetic dipoles align with the magnetic field in each type of substance
- Define hysteresis and magnetic susceptibility, which determines the type of magnetic material

Why are certain materials magnetic and others not? And why do certain substances become magnetized by a field, whereas others are unaffected? To answer such questions, we need an understanding of magnetism on a microscopic level.

Within an atom, every electron travels in an orbit and spins on an internal axis. Both types of motion produce current loops and therefore magnetic dipoles. For a particular atom, the net magnetic dipole moment is the vector sum of the magnetic dipole moments. Values of μ for several types of atoms are given in **Table 12.1**. Notice that some atoms have a zero net dipole moment and that the magnitudes of the nonvanishing moments are typically $10^{-23}\ \text{A} \cdot \text{m}^2$.

Atom	Magnetic Moment $\left(10^{-24}\ \text{A} \cdot \text{m}^2\right)$
H	9.27
He	0
Li	9.27
O	13.9
Na	9.27
S	13.9

Table 12.1 Magnetic Moments of Some Atoms

A handful of matter has approximately 10^{26} atoms and ions, each with its magnetic dipole moment. If no external magnetic field is present, the magnetic dipoles are randomly oriented—as many are pointed up as down, as many are pointed east as west, and so on. Consequently, the net magnetic dipole moment of the sample is zero. However, if the sample is placed in a magnetic field, these dipoles tend to align with the field (see **Equation 12.14**), and this alignment determines how the sample responds to the field. On the basis of this response, a material is said to be either paramagnetic, ferromagnetic, or diamagnetic.

In a **paramagnetic material**, only a small fraction (roughly one-third) of the magnetic dipoles are aligned with the applied field. Since each dipole produces its own magnetic field, this alignment contributes an extra magnetic field, which enhances the applied field. When a **ferromagnetic material** is placed in a magnetic field, its magnetic dipoles also become aligned;

furthermore, they become locked together so that a permanent magnetization results, even when the field is turned off or reversed. This permanent magnetization happens in ferromagnetic materials but not paramagnetic materials. **Diamagnetic materials** are composed of atoms that have no net magnetic dipole moment. However, when a diamagnetic material is placed in a magnetic field, a magnetic dipole moment is directed opposite to the applied field and therefore produces a magnetic field that opposes the applied field. We now consider each type of material in greater detail.

Paramagnetic Materials

For simplicity, we assume our sample is a long, cylindrical piece that completely fills the interior of a long, tightly wound solenoid. When there is no current in the solenoid, the magnetic dipoles in the sample are randomly oriented and produce no net magnetic field. With a solenoid current, the magnetic field due to the solenoid exerts a torque on the dipoles that tends to align them with the field. In competition with the aligning torque are thermal collisions that tend to randomize the orientations of the dipoles. The relative importance of these two competing processes can be estimated by comparing the energies involved. From Equation 12.14, the energy difference between a magnetic dipole aligned with and against a magnetic field is $U_B = 2\mu B$. If $\mu = 9.3 \times 10^{-24}\,\text{A} \cdot \text{m}^2$ (the value of atomic hydrogen) and $B = 1.0$ T, then

$$U_B = 1.9 \times 10^{-23}\,\text{J}.$$

At a room temperature of $27\,°\text{C}$, the thermal energy per atom is

$$U_T \approx kT = (1.38 \times 10^{-23}\,\text{J/K})(300\,\text{K}) = 4.1 \times 10^{-21}\,\text{J},$$

which is about 220 times greater than U_B. Clearly, energy exchanges in thermal collisions can seriously interfere with the alignment of the magnetic dipoles. As a result, only a small fraction of the dipoles is aligned at any instant.

The four sketches of Figure 12.23 furnish a simple model of this alignment process. In part (a), before the field of the solenoid (not shown) containing the paramagnetic sample is applied, the magnetic dipoles are randomly oriented and there is no net magnetic dipole moment associated with the material. With the introduction of the field, a partial alignment of the dipoles takes place, as depicted in part (b). The component of the net magnetic dipole moment that is perpendicular to the field vanishes. We may then represent the sample by part (c), which shows a collection of magnetic dipoles completely aligned with the field. By treating these dipoles as current loops, we can picture the dipole alignment as equivalent to a current around the surface of the material, as in part (d). This fictitious surface current produces its own magnetic field, which enhances the field of the solenoid.

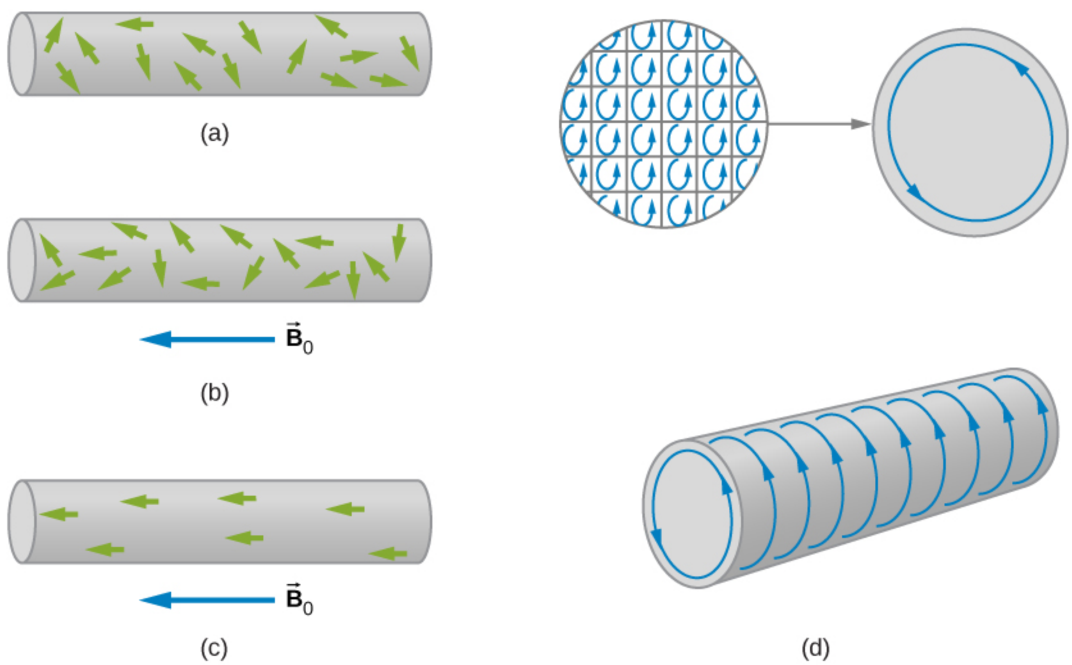

Figure 12.23 The alignment process in a paramagnetic material filling a solenoid (not shown). (a) Without an applied field, the magnetic dipoles are randomly oriented. (b) With a field, partial alignment occurs. (c) An equivalent representation of part (b). (d) The internal currents cancel, leaving an effective surface current that produces a magnetic field similar to that of a finite solenoid.

We can express the total magnetic field $\vec{\mathbf{B}}$ in the material as

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \vec{\mathbf{B}}_m, \tag{12.34}$$

where $\vec{\mathbf{B}}_0$ is the field due to the current I_0 in the solenoid and $\vec{\mathbf{B}}_m$ is the field due to the surface current I_m around the sample. Now $\vec{\mathbf{B}}_m$ is usually proportional to $\vec{\mathbf{B}}_0$, a fact we express by

$$\vec{\mathbf{B}}_m = \chi \vec{\mathbf{B}}_0, \tag{12.35}$$

where χ is a dimensionless quantity called the **magnetic susceptibility**. Values of χ for some paramagnetic materials are given in **Table 12.2**. Since the alignment of magnetic dipoles is so weak, χ is very small for paramagnetic materials. By combining **Equation 12.34** and **Equation 12.35**, we obtain:

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_0 + \chi \vec{\mathbf{B}}_0 = (1+\chi)\vec{\mathbf{B}}_0. \tag{12.36}$$

For a sample within an infinite solenoid, this becomes

$$B = (1+\chi)\mu_0 nI. \tag{12.37}$$

This expression tells us that the insertion of a paramagnetic material into a solenoid increases the field by a factor of $(1+\chi)$. However, since χ is so small, the field isn't enhanced very much.

The quantity

$$\mu = (1+\chi)\mu_0. \tag{12.38}$$

is called the magnetic permeability of a material. In terms of μ, **Equation 12.37** can be written as

$$B = \mu nI \tag{12.39}$$

for the filled solenoid.

Paramagnetic Materials	χ	Diamagnetic Materials	χ
Aluminum	2.2×10^{-5}	Bismuth	-1.7×10^{-5}
Calcium	1.4×10^{-5}	Carbon (diamond)	-2.2×10^{-5}
Chromium	3.1×10^{-4}	Copper	-9.7×10^{-6}
Magnesium	1.2×10^{-5}	Lead	-1.8×10^{-5}
Oxygen gas (1 atm)	1.8×10^{-6}	Mercury	-2.8×10^{-5}
Oxygen liquid (90 K)	3.5×10^{-3}	Hydrogen gas (1 atm)	-2.2×10^{-9}
Tungsten	6.8×10^{-5}	Nitrogen gas (1 atm)	-6.7×10^{-9}

Table 12.2 Magnetic Susceptibilities *Note: Unless otherwise specified, values given are for room temperature.

Paramagnetic Materials	χ	Diamagnetic Materials	χ
Air (1 atm)	3.6×10^{-7}	Water	-9.1×10^{-6}

Table 12.2 Magnetic Susceptibilities *Note: Unless otherwise specified, values given are for room temperature.

Diamagnetic Materials

A magnetic field always induces a magnetic dipole in an atom. This induced dipole points opposite to the applied field, so its magnetic field is also directed opposite to the applied field. In paramagnetic and ferromagnetic materials, the induced magnetic dipole is masked by much stronger permanent magnetic dipoles of the atoms. However, in diamagnetic materials, whose atoms have no permanent magnetic dipole moments, the effect of the induced dipole is observable.

We can now describe the magnetic effects of diamagnetic materials with the same model developed for paramagnetic materials. In this case, however, the fictitious surface current flows opposite to the solenoid current, and the magnetic susceptibility χ is negative. Values of χ for some diamagnetic materials are also given in Table 12.2.

Water is a common diamagnetic material. Animals are mostly composed of water. Experiments have been performed on frogs (https://openstaxcollege.org/l/21frogs) and mice (https://openstaxcollege.org/l/21mice) in diverging magnetic fields. The water molecules are repelled from the applied magnetic field against gravity until the animal reaches an equilibrium. The result is that the animal is levitated by the magnetic field.

Ferromagnetic Materials

Common magnets are made of a ferromagnetic material such as iron or one of its alloys. Experiments reveal that a ferromagnetic material consists of tiny regions known as **magnetic domains**. Their volumes typically range from 10^{-12} to $10^{-8}\,m^3$, and they contain about 10^{17} to 10^{21} atoms. Within a domain, the magnetic dipoles are rigidly aligned in the same direction by coupling among the atoms. This coupling, which is due to quantum mechanical effects, is so strong that even thermal agitation at room temperature cannot break it. The result is that each domain has a net dipole moment. Some materials have weaker coupling and are ferromagnetic only at lower temperatures.

If the domains in a ferromagnetic sample are randomly oriented, as shown in Figure 12.24, the sample has no net magnetic dipole moment and is said to be unmagnetized. Suppose that we fill the volume of a solenoid with an unmagnetized ferromagnetic sample. When the magnetic field $\vec{\mathbf{B}}_0$ of the solenoid is turned on, the dipole moments of the domains rotate so that they align somewhat with the field, as depicted in Figure 12.24. In addition, the aligned domains tend to increase in size at the expense of unaligned ones. The net effect of these two processes is the creation of a net magnetic dipole moment for the ferromagnet that is directed along the applied magnetic field. This net magnetic dipole moment is much larger than that of a paramagnetic sample, and the domains, with their large numbers of atoms, do not become misaligned by thermal agitation. Consequently, the field due to the alignment of the domains is quite large.

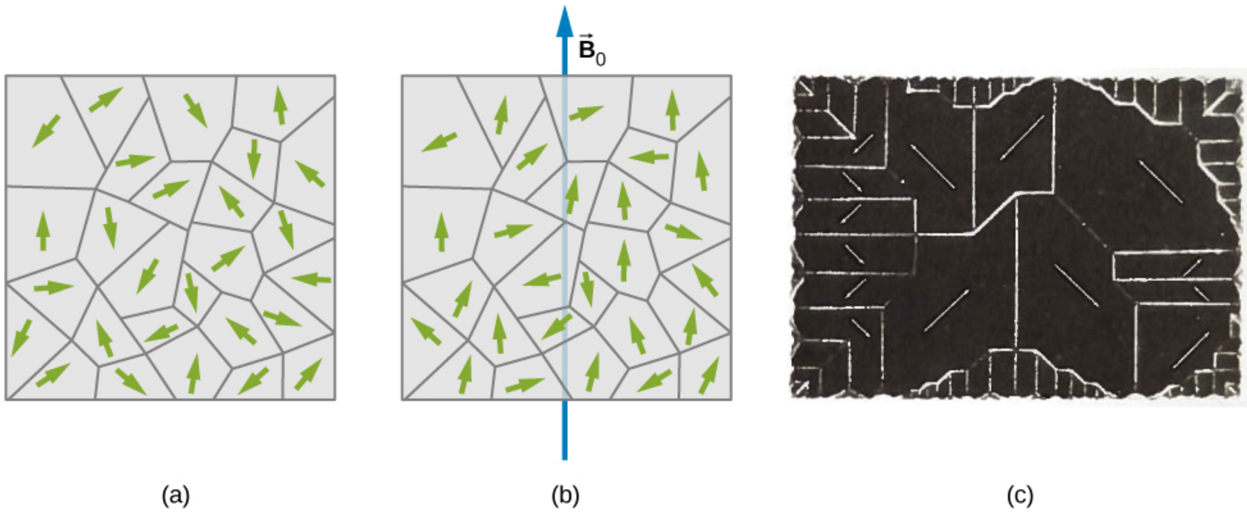

Figure 12.24 (a) Domains are randomly oriented in an unmagnetized ferromagnetic sample such as iron. The arrows represent the orientations of the magnetic dipoles within the domains. (b) In an applied magnetic field, the domains align somewhat with the field. (c) The domains of a single crystal of nickel. The white lines show the boundaries of the domains. These lines are produced by iron oxide powder sprinkled on the crystal.

Besides iron, only four elements contain the magnetic domains needed to exhibit ferromagnetic behavior: cobalt, nickel, gadolinium, and dysprosium. Many alloys of these elements are also ferromagnetic. Ferromagnetic materials can be described using **Equation 12.34** through **Equation 12.39**, the paramagnetic equations. However, the value of χ for ferromagnetic material is usually on the order of 10^3 to 10^4, and it also depends on the history of the magnetic field to which the material has been subject. A typical plot of *B* (the total field in the material) versus B_0 (the applied field) for an initially unmagnetized piece of iron is shown in **Figure 12.25**. Some sample numbers are (1) for $B_0 = 1.0 \times 10^{-4}\,\text{T}, B = 0.60\,\text{T},$ and $\chi = \left(\frac{0.60}{1.0 \times 10^{-4}}\right) - 1 \approx 6.0 \times 10^3$; (2) for $B_0 = 6.0 \times 10^{-4}\,\text{T}, B = 1.5\,\text{T},$ and $\chi = \left(\frac{1.5}{6.0 \times 10^{-4}}\right) - 1 \approx 2.5 \times 10^3$.

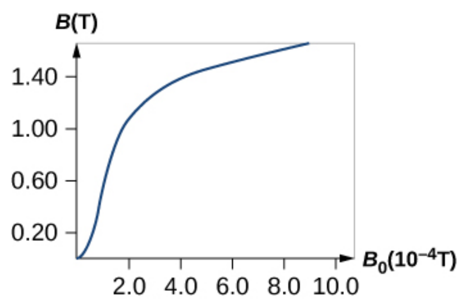

Figure 12.25 (a) The magnetic field *B* in annealed iron as a function of the applied field B_0.

When B_0 is varied over a range of positive and negative values, *B* is found to behave as shown in **Figure 12.26**. Note that the same B_0 (corresponding to the same current in the solenoid) can produce different values of *B* in the material. The magnetic field *B* produced in a ferromagnetic material by an applied field B_0 depends on the magnetic history of the material. This effect is called **hysteresis**, and the curve of **Figure 12.26** is called a hysteresis loop. Notice that *B* does not disappear when $B_0 = 0$ (i.e., when the current in the solenoid is turned off). The iron stays magnetized, which means that it has become a permanent magnet.

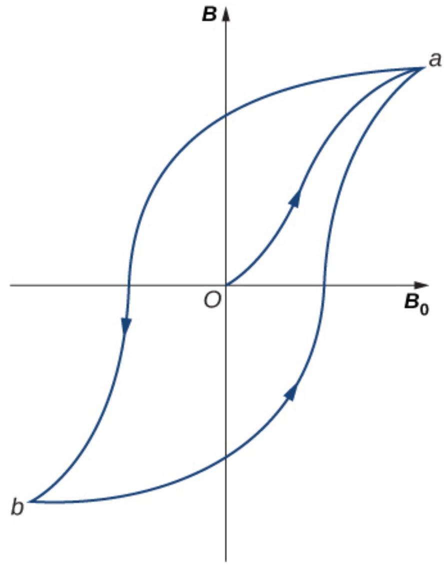

Figure 12.26 A typical hysteresis loop for a ferromagnet. When the material is first magnetized, it follows a curve from 0 to *a*. When B_0 is reversed, it takes the path shown from *a* to *b*. If B_0 is reversed again, the material follows the curve from *b* to *a*.

Like the paramagnetic sample of **Figure 12.23**, the partial alignment of the domains in a ferromagnet is equivalent to a current flowing around the surface. A bar magnet can therefore be pictured as a tightly wound solenoid with a large current circulating through its coils (the surface current). You can see in **Figure 12.27** that this model fits quite well. The fields of the bar magnet and the finite solenoid are strikingly similar. The figure also shows how the poles of the bar magnet are identified. To form closed loops, the field lines outside the magnet leave the north (N) pole and enter the south (S) pole, whereas inside the magnet, they leave S and enter N.

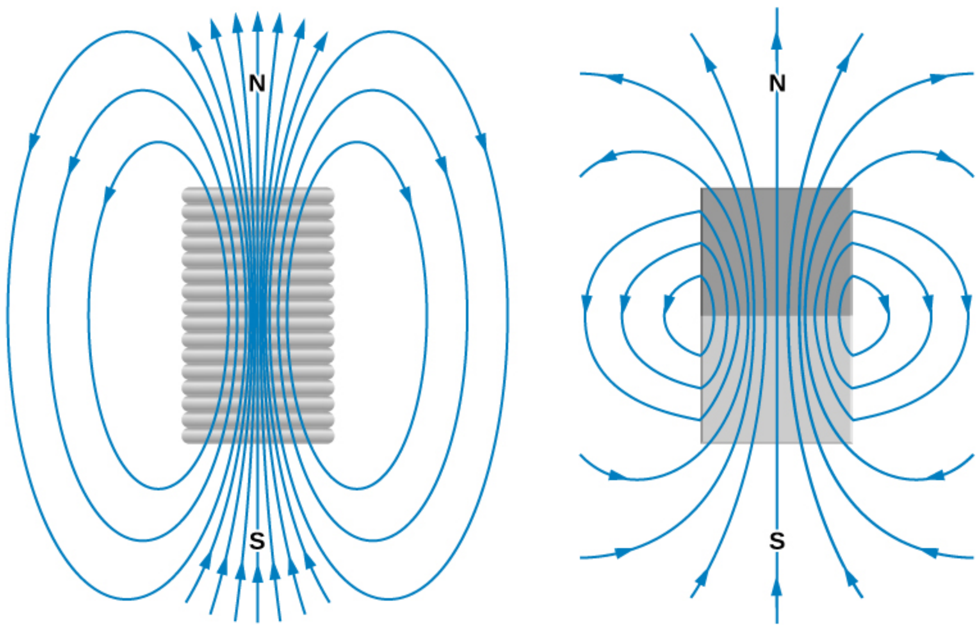

Figure 12.27 Comparison of the magnetic fields of a finite solenoid and a bar magnet.

Ferromagnetic materials are found in computer hard disk drives and permanent data storage devices (**Figure 12.28**). A material used in your hard disk drives is called a spin valve, which has alternating layers of ferromagnetic (aligning with the external magnetic field) and antiferromagnetic (each atom is aligned opposite to the next) metals. It was observed that a significant change in resistance was discovered based on whether an applied magnetic field was on the spin valve or not. This large change in resistance creates a quick and consistent way for recording or reading information by an applied current.

Figure 12.28 The inside of a hard disk drive. The silver disk contains the information, whereas the thin stylus on top of the disk reads and writes information to the disk.

Example 12.10

Iron Core in a Coil

A long coil is tightly wound around an iron cylinder whose magnetization curve is shown in **Figure 12.25**. (a) If $n = 20$ turns per centimeter, what is the applied field B_0 when $I_0 = 0.20\ \text{A}$? (b) What is the net magnetic field for this same current? (c) What is the magnetic susceptibility in this case?

Strategy

(a) The magnetic field of a solenoid is calculated using **Equation 12.28**. (b) The graph is read to determine the net magnetic field for this same current. (c) The magnetic susceptibility is calculated using **Equation 12.37**.

Solution

a. The applied field B_0 of the coil is

$$\begin{aligned} B_0 &= \mu_0 n I_0 = (4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})(2000/\text{m})(0.20\ \text{A}) \\ B_0 &= 5.0 \times 10^{-4}\ \text{T}. \end{aligned}$$

b. From inspection of the magnetization curve of **Figure 12.25**, we see that, for this value of B_0, $B = 1.4\ \text{T}$. Notice that the internal field of the aligned atoms is much larger than the externally applied field.

c. The magnetic susceptibility is calculated to be

$$\chi = \frac{B}{B_0} - 1 = \frac{1.4\ \text{T}}{5.0 \times 10^{-4}\ \text{T}} - 1 = 2.8 \times 10^3.$$

Significance

Ferromagnetic materials have susceptibilities in the range of 10^3 which compares well to our results here. Paramagnetic materials have fractional susceptibilities, so their applied field of the coil is much greater than the magnetic field generated by the material.

12.8 Check Your Understanding Repeat the calculations from the previous example for $I_0 = 0.040 \text{ A}$.